Modern Atomic Theory

Quantum Mechanics

The Wave Nature of Matter Page [1 of 3]

We've got a big problem. We know from the hydrogen emissions spectrum and from the absorption spectrum from the sun that energy is quantized in a hydrogen atom. But the electrons in the hydrogen atom have specific allowed energies. But, our model, the Bohr atom, successfully explained why that would be true ultimately fails because of the Heisenberg Uncertainty Principle. In other words, we can't determine both position and momentum of the electron at the same time. So, it can't be traveling in a specified orbit. So, we have to throw out the idea entirely that an electron is orbiting around the nucleus, like a planet is around the sun, and come up with some different idea.

But, what do we do? In order to take the next conceptual leap in quantum mechanics, we have to recognize that just like light as particle properties as well as wave properties that matter too can have particle properties and wave properties. The person that originally postulated this was De Broglie who said particles have wave properties. We've heard that before. Mathematically he said that the wavelength of those associated with that particle is Planck's constant over the momentum. What he is saying in essence is that if you have two particles of equal mass traveling at different speeds that the slower particle will have a bigger wavelength than the fast particle will. Or, alternatively, if you have a large particle and a small particle traveling at the same speed the large particle will have a much smaller wavelength associated with it.

Now, what does this have to do with anything? We'll find, ultimately, that by having wave properties, in particular by the electrons having wave properties; that will ultimately place some restrictions on what their positions can be and most importantly, what their energies can be. Now, before we get there, let's look at a little more detail of what De Broglie is saying.

Let's actually calculate what the wavelength would be of an electron traveling at, kind of, a typical speed. So, let's take an electron with a velocity of 10^6 m/s and solve for the De Broglie wavelength. The wavelength is going to be, again, Planck's constant over its momentum. Plugging in Planck's constant and momentum, which you recall is mass times velocity in classical mechanics, mass of the electron and velocity we end up with. And, by the way, as you are canceling units here, remember that joules have units of kilograms meters squared per second squared. So, you will cancel out all your units and you will be left just with meters, in this case. In particular, that electron has 7.3×10^{-10} m wavelength, or, in other words, 7.3 angstroms. Now, here is the important point with angstroms is on the same scale as the atom itself that the electron is in. We are going to see that has profound consequences.

To understand the profound consequences, we are going to talk about standing waves. But before I do that I really should show you the experimental evidence that, in fact, De Broglie was right, because he just theorized this relationship.

It wasn't until 1927 that Davisson and Germer did an experiment showing that particles have wave properties. Now, to describe this experiment I need to back up one more step and talk to you a little bit about waves and something that we are more familiar with like water. So, imagine that I had a propagating wave front in water. And I passed that water through two slits, two new openings. I then have, in a sense, a source of two different waves that propagate out, but they are in phase with each other. In other words, each wave hits those two slits at the same time. So, the two waves that are formed come out in phase with each other.

What I am going to find out is that depending on where we are in the water out here, we'll either be in a position where the two peaks coming from the two different sources are in phase with each other, in which case they constructively interfere. And, I end up with a peak twice as big as either of these waves by themselves. Or, we have positions where they destructively interfere. In other words, where one wave at a given position happens to be exactly out of phase now with the other positions simply because the distance from here is not the same as the distance from here. So, it took this guy a little longer to get here. So, they are out of phase with each other. As a result, I 'm going to see what is referred to as an interference pattern. Where, again, the waves constructively interfere at one position and then destructively interfere at another. That is going to be the hallmark feature of wave properties. And, if indeed particles have wave properties, we should see an interference pattern for particles as well.

Now, we need very, very small slits. Remember, we calculated the wavelength of electrons for the example we had was on the order of angstroms. I need to have an extremely narrow slit now in order to pass electrons through. Smaller than anything we could fabricate. But, nature has done that for us. When nature makes a crystal, as we've seen, the spacing between the atoms in that crystal are on the order of angstroms, the size of atoms. So, in a crystal of, let's say, magnesium oxide the separations between these atoms are on the order of angstroms, which is on the order of the wavelength that we expect, by De Broglie at least, for the electrons to have.

So, Davisson and Germer passed electrons through crystal, which I just on this cartoon show in two dimensions like this. And, sure enough, what they saw was an interference pattern on the photographic plate that captured these electrons. What I mean by that is there were some places where there was a very large collection of where the electrons hit. And, other places where the electrons seemingly were forbidden. The reason those electrons were forbidden was because the wave properties of the electrons destructively interfered. And, so there was no chance of the electrons arriving there.

Experimentally what this looks like, if we actually look at the real data, is because a crystal is three dimensional, rather then the analogy I used being a one dimensional or a two dimensional example, we actually see these rings coming out from the crystal. In fact, this is a very useful technique these days to determine spacing between atoms in unknown materials. The point I want to make right now is that by seeing these rings, in other words, places where we have positive and negative interference, constructive and destructive interference, that demonstrated absolutely that De Broglie was absolutely right. That particles do, in fact, have wave properties. And, in fact, the De Broglie's relationship was exactly right in calculating what the interference pattern should be. In other words, what the wavelength, indeed, would have to be to make this particular interference pattern with that particular spacing. So, this was a huge success. Of course, De Broglie was pretty delighted that his theory was proven to be correct.

The next step is for us to look at the idea of a standing wave. We were talking about a propagating wave. Bear with me here. We will tie this back together in a moment. But, we have to talk a little bit about what a standing wave is. To do that I'm going to use this guitar to help us get an idea of that. So, let me describe the important part of a guitar as far as I am concerned. I have a string tied together at two ends. I am restricting the ends of this string to be nonmoving, and have no motion at the edges. And, I have restricted the length of this string. As long as I am careful not to hold the string down against any of the frets, I'm not changing the length of the string. Now, when I pluck the string let's listen carefully and listen to the sound that it makes. I'm going to do this again. Listen to the note.

Now, I'm going to do the exact same thing but this time I am going to very lightly place my finger exactly halfway between the string. And, you are going to hear a different note even after I take my finger off. So, notice even when I'm not touching the string, the frequency that I get out of the string is now different. I can go to another position and play a third note. That is getting a little hard to hear now. But, again, notice that every time I take my finger off the string and I still get a different note.

Let me show you what I am actually doing. When I pluck this string without touching the string at all, I am playing what is called the fundamental frequency of that string. The string is just vibrating up and down like this. However, when I touch the middle of the string, I am canceling out that particular vibration, I am disallowing it by holding that string down. But, I don't disallow this wavelength. In other words, since this particular wavelength has zero at either end, in other words, it is restricted at the end still. And, it is restricted exactly in the middle by me holding my finger right in the middle; I don't damp out this wave. So, what you heard, and I'll do it once more for you here, the difference between that and that, the first harmonic and the fundamental, is the difference in these two wavelengths.

So, this length of guitar string supports this wavelength and this wavelength. And, in fact, let's go ahead and do this wavelength as well. I'm going to hold it now at a third of the position. That is the second harmonic. So I can support this note, this note, this note, but I can't support something in between. So, in other words, instead of holding my finger right in the middle, I just move it a little bit and now I can't play anything. It's only at right at that position that this particular wavelength fits over that length of string.

The point in all of this is that this standing wave is restricted to support, if you will, only certain frequencies of this string. In other words, the string's frequencies or wavelengths are quantized. They are quantized because I've restricted the length of the string. If I had an infinitely long string, I could play any note I wanted on that string and all would be supported. But, because I restricted the string, I've quantized the allowed frequencies that I can play.

Now, it is quite a leap to go from this one-dimensional example to the three dimensions of an atom. But, the idea is going to be similar. I will try, maybe unsuccessfully, to show you with this other drawing in two dimensions where we are going with this. And, that is that if I have an electron with a particular energy that determines its wavelength. If I determine its wavelength and I'm restricting that electron to be around a nucleus the wavelength must come around and constructively interfere with itself. If instead I have a wavelength that doesn't exactly fit into whatever this pattern is the wavelength will destructively interfere with itself. And, so, as I say that particular energy would not be supported, in other words the energy of the electron is going to be quantized because it has wave properties associated with it. And, we are trying to restrict that electron around a nucleus. So, what ends up happening is these wave properties will destructively interfere unless we have specifically allowed energies of the electron. Therefore, our electron energy is quantized.

Returning finally to our analogy of the guitar string. We said that by restricting the guitar string length that posed restrictions on the frequencies that we could play on that guitar string. In other words, the frequencies were quantized. Only certain allowed values would be supported by that string. My analogy then by acknowledging that electrons have wave properties in an atom that results in energies having only certain allowed energies. In other words, the energies of the electron are quantized.