An Introduction to Matter and Measurement

Mathematics of Chemistry

Scientic (Exponential) Notation Page [1 of 2]

Scientists have their own special language, and I'm going to share one of the secrets with you know, how we write down really, really, really big numbers. It's a shorthand notation that includes all of the information, but it saves a lot of paper. And that technique is called writing a number in scientific notation.

So here's a number that some of you may recognize already and, if you don't recognize it, it's no big deal, but it is a very big number. It's 602213700, followed by 15 more zeros. And, obviously, if every time you had to write down this number and, for reasons that'll become clear, it'll be important to write down this number several times in your study of chemistry, you'd use a lot of paper. So rather than write it this way, what we're going to do is we're going to forget about all the zeros. Not forget about them, but we're going to use a shorthand notation for taking care of all of the zeros. And the reason why we can do this is because the zeros aren't really significant numbers anyway, they're placeholders. And so what we do is we express numbers, either very large numbers or very small numbers, and both positive and negative numbers, by writing them in two pieces. And the first piece is the piece that includes all of the non-zero digits and maybe some zeros that happen to be interspersed in there. This part of the number is going to be greater than or equal to one and less than ten. So, in other words, there will be a number, followed by a decimal point, followed by a bunch more numbers. It's possible that you wouldn't have the decimal point, that it would just be a single digit, but the point is that digit is going to be one through nine. And then we have a multiplication sign, and then we'd follow it by ten to some power, where this power tells us how many zeros we have to include in order to get this number to be exactly the same as this number. And the way to remember whether this number should be positive or negative is, first of all, big numbers have a positive exponent and small numbers, numbers that are less than one, have a negative exponent. And to go from this number to this number, recognize that we go from this point, so there's an implied decimal point at the end of the zeros here, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three. In order to get the decimal point between the six and the zero, we have to move the decimal point twenty-three times to the left. And this twenty-three tells us that's how many places we had to move it in order to get it into what we call exponential notation. Now, one of the really good benefits of exponential notation is that every digit in this part of the number is a significant figure. So you have to consider every single digit here as significant.

Now, let's look at some numbers and convert them into scientific notation and make sure that we understand how we go about the process of getting from a number that's just written out with all the zeros to a number that's in scientific or exponential notation.

25.0, remember, we want a number that's between one and 9.9999999, greater than or equal to one or less than ten. So to go from 25.0 to something in exponential notation, we have to move the decimal place one place to the left, which means that we put a one here, a positive one, and that number is significant, so it's 2.50×10^1. Here we have a number that's less than one, and so we're going to have to move the decimal place the other way to get it into scientific notation, one, two, three, four. So it's 3.71, and then 10^{-4}, because we had to move the decimal place to the right to get it into scientific notation. And 1,376, again, we're moving it to the left, one, two, three. So there's a 10^3. And here's .33167, so we'd have to move that one, one place to the right. And so we have a minus one here.

Now, again, one of the great advantages of scientific notation is that the number of significant figures is unambiguous. So 100 is somewhat ambiguous. Does it have one significant figure? Does it have two significant figures? Does it have three significant figures? Some conventions say that, if it has a period here, a decimal place, then it has three significant figures. And if it doesn't have a decimal place, then it only has one significant figure. And that's a reasonable way to do it, but it's somewhat ambiguous. It's entirely unambiguous if we express 100 in scientific notation, because we can either express it as 1×10^2, which has one significant figure, or 1.0×10^2, which has two significant figures, or 1.00×10^2, which has three significant figures. If we want to express it to four, or five, or six significant figures, the point is, we'd just be putting significant zeros after the decimal place and the placeholder part, the 10^2, goes along for the ride.

Now, when we're doing arithmetic with numbers in scientific notation, if you're using your calculator, you really can ignore the rest of this lecture, because your calculator is going to be doing what I'm telling you. You can do longhand, if you don't happen to have a calculator, or if your teacher doesn't allow you to use calculators. We know, for instance, that 3.94 + 0.67 = 4.61. If this is written in scientific notation, it would be 3.41, so you don't typically write 10^0 for numbers that are between one and 9.99999. You just would write 3.94 and forget about the fact that 10^0 is just one. And then if you're adding this number written in scientific notation, scientific notation would look like that. And what you have to do is you have to convert them both to have the same power of ten. So the exponentials must be

An Introduction to Matter and Measurement

Mathematics of Chemistry

Scientic (Exponential) Notation Page [2 of 2]

same when you want to add them. And so, if we, for instance, converted this back into, excuse me, put it in with an exponent of zero, it means that we would convert this number into this. So this number is equal to this number, 0.67×10^{0}, just as this is 3.94×10^{0}. So we can convert this number to here, and then we can get to the answer. And remember that, when you're considering the number of significant figures in an addition problem, it is if we have two figures past the decimal point in this number and two in this number, then the sum is going to have two significant figures past the decimal point. Whichever number has the fewer number of places past the decimal point, that's what determines the number of significant figures in a sum or a difference.

The other way to do this is to convert them both into numbers that have an exponent of minus one. And, of course, this is not, this number is not in scientific notation. But now that they both have an exponent of minus one, we can add these two parts and we get 46.1×10^{-1}. So when the exponents are the same, then it comes along for the ride. And then converting this back into scientific notation, we get to the same answer, 4.61.

When we're multiplying numbers, recognize that $10^{a} \times 10^{b} = 10^{a+b}$, where this is the exponent. So $3.94 \times 10^{2} \times 6.7 \times 10^{-3}$, we don't have to do this conversion into a common exponential, but we do have to group together the part that isn't a power of ten. So, for instance, we group 3.94 and 6.7. And then we group the parts that are a power of ten and use the fact that $10^{a} \times 10^{b} = 10^{a+b}$. So we would add the exponents to a minus three, and then we would multiply these parts out, 3.94×6.7, which, if you work it out on a calculator, looks like 26.398. Then you have to put this part into the correct number of significant figures, which, in this case, is going to be two, because there are three significant figures in 3.94, but only two in 6.7. So to the correct number of significant figures, this product is 26×10^{-1}. And then, correcting it to scientific notation, we move the decimal place one place to the right and that cancels out the minus one here. So the answer is 2.6. Plug it into your calculator and you'll see that the answer is, in fact, 2.6 with a bunch of other numbers, but rounding, considering the number of significant figures, the answer is 2.6.

And finally, we have to consider powers and roots and, once again, with powers and roots, if you're using your electronic calculator, it's not really an issue. But 10^{a} raised to the B power is 10^{ab}. And so if we have a number that's $R \times 10^{t}$ raised to the B power, so this whole number raised to the B power, it is $R^{b} \times 10^{tb}$. And so, for instance, $6.7 \times 10^{-1^{4}} = 6.7^{4}$, and then 10^{tb}, which, in this case, is -1×4. Now, we multiply this product, and this part is the part that has the significant figures, and I've expressed it with four significant figures here. So $(6.7)^{4}$, which is $6.7 \times 6.7 \times 6.7 \times 6.7 = 2{,}015$ and a bunch of other digits, but this is already too many significant figures. When we round to the correct number of significant figures, which is two, because we're multiplying, we get 2.0, and then 10^{-4}, excuse me, we get back, this is 2,015, so we get back three orders of magnitude or three powers of ten. So those three powers of ten cancel with most of the four powers of ten here, so we get a minus one. Let me express that; $2.015 \times 10^{3} \times 10^{-4}$. So 2,015 is 2.015×10^{3}, and then there's a 10^{-4}, so the three and the four from the previous page become a minus one. And then, when we round this to the correct number of significant figures, we get 2.0×10^{-1}.

And to take square roots or n-th roots, this is the b-th root of the number $r \times 10^{t}$. It's R to the one over b power. Remember that a b-th root is a number raised to the power one over b times ten to the t over b. This is just an extension of what we had on the previous page. Another way to write this is $r \times 10^{t}$, all raised to the one over b power. And so that's r to the one over b times 10 to the t over b. And so if we wanted to take the square root of 9.67×10^{-4}, it's 9.67 taken to the half-power times 10^{-4} raised to the half-power, and that's 3.11 to the correct number of significant figures and 10 to the minus four over two, which is 10^{-2}.

So again, all of the stuff about addition and multiplication, the only thing you really need to worry about is rounding to the correct number of significant figures. Your calculator is going to take care of the rest of it. But, on the other hand, it would be great if what the calculator is doing was not a great mystery to you; that you were actually aware of the arithmetic and aware of the algebra and what exactly is going on. Other than that, scientific notation is a way to express really big numbers, numbers much, much larger than one or much, much smaller than one, in a convenient way that makes it really crystal clear how big the number is without having to count all of the zeros.